Trigonometry & Calculus

Grade Level:	11, 12
Length:	1 Year
Period(s) Per Day:	1
Credit:	1
Credit Requirement Fulfilled:	Mathematics

Course Description

Students will investigate the underlying principles and applications of Trigonometry. This course will review analytic geometry (conic sections), study trigonometric functions and their use in modeling real-world phenomena, study trigonometric identities, study the principles and concepts of the derivative, and apply differential calculus to solve problems in a variety of areas including, but not limited to, physics and business. **This course can be dual credit with MATH 112 within the Montana University System.**

Theme Samples

1. Functions, Transformations, and Inverses
2. Trigonometric Functions
3. Analytic Trigonometry
4. Polar Coordinates and Parametric Equations
5. Vectors
6. Conic Sections
7. Exponential and Logarithmic Functions
8. Limits and Continuity
9. Derivatives
10. Integrals

Course Objectives and Expectations

1. To review functions and graphs.
2. To solve right triangles and oblique triangles.
3. To manipulate and graph trigonometric functions, inverse trigonometric functions, and composite functions.
4. To solve trigonometric equations.
5. To manipulate and graph parametric equations.
6. To understand two and three dimensional geometry.
7. To analyze the geometric properties of conic sections.
8. To manipulate, solve, and graph exponential and logarithmic functions.
9. To define and calculate limits of function values.
10. To understand derivatives, how the work, and discover differential calculus.

Pacing	***Montana Common Core Standard***
Semester 1	
Unit 1- Functions, Transformations, and Inverses	F-IF, F-BF
Unit 2- Trigonometric Functions (Unit Circle Approach)	F-TF
Unit 3- Trigonometric Functions (Right Triangle Approach)	F-TF, G-SRT
Unit 4- Analytic Trigonometry	F-TF
Unit 5- Polar Coordinates and Parametric Equations	(Advanced, no CCSS)
Unit 6- Vectors in Two and Three Dimensions	N-VM

1st Semester

- Functions, Transformations, and Inverses
  - A. Coordinate Geometry
  - B. Lines
  - C. What is a Function?
  - D. Graphs of Functions
  - E. Getting Information from the Graph of a Function
  - F. Transformations of Functions
  - G. Combining Functions
  - H. One-to-One Functions and Their Inverses
- Trigonometric Functions (Unit Circle Approach)
  - A. The Unit Circle
  - B. Trigonometric Functions of Real Numbers
  - C. Trigonometric Graphs
  - D. More Trigonometric Graphs
  - E. Inverse Trigonometric Functions and Their Graphs
  - F. Modeling Harmonic Motion
- Trigonometric Functions (Right Triangle Approach)
  - A. Angle Measure
  - B. Trigonometry of Right Triangles
  - C. Trigonometric Functions of Angles
  - D. Inverse Trigonometric Functions and Right Triangles
  - E. The Law of Sines
  - F. The Law of Cosines
- Analytic Trigonometry
  - A. Trigonometric Identities
  - B. Addition and Subtraction Formulas
  - C. Double-Angle, Half-Angle, and Product-Sum Formulas
  - D. Basic Trigonometric Equations
  - E. More Trigonometric Equations
- Polar Coordinates and Parametric Equations
  - A. Polar Coordinates

B. Graphs of Polar Equations
C. Polar Form of Complex Numbers; De Moivre's Theorem
D. Plane Curves and Parametric Equations

Vectors in Two and Three Dimensions

A. Vectors in Two Dimensions
B. The Dot Product
C. Three-Dimensional Coordinate Geometry
D. Vectors in Three Dimensions
E. The Cross Product
F. Equations of Lines and Planes

Semester 2

Unit 7- Conic Sections	G-GPE
Unit 8- Exponential and Logarithmic Functions	F-LE
Unit 9- Limits and Continuity	(Advanced, no CCSS)
Unit 10- Derivatives	(Advanced, no CCSS)

2nd Semester

Conic Sections

A. Parabolas
B. Ellipses
C. Hyperbolas
D. Shifted Conics
E. Rotation of Axes
F. Polar Equations of Conics

Exponential and Logarithmic Functions

A. Exponential Functions
B. The Natural Exponential Function
C. Logarithmic Functions
D. Laws of Logarithms
E. Exponential and Logarithmic Equations
F. Modeling with Exponential and Logarithmic Functions
G. Damped Harmonic Motion

Limits and Continuity

A. Rates of Change and Limits
B. Limits Involving Infinity
C. Continuity
D. Rates of Change and Tangent Lines

Derivatives

A. Derivative of a Function

B. Differentiability
C. Rules for Differentiation
D. Velocity and Other Rates of Change
E. Derivatives of Trigonometric Functions
F. Chain Rule
G. Implicit Differentiation
H. Derivatives of Inverse Trigonometric Functions
I. Derivatives of Exponential and Logarithmic Functions

Timeline

Unit 1	(1 ½ week to cover)
Unit 2	(3 weeks to cover)
Unit 3	(2 ½ weeks to cover)
Unit 4	(3 weeks to cover)
Unit 5	(2 ½ weeks to cover)
Unit 6	(2 ½ weeks to cover)
Unit 7	(2 ½ weeks to cover)
Unit 8	(3 weeks to cover)
Unit 9	(3 ½ weeks to cover)
Unit 10	(3 ½ weeks to cover)

Montana Standards for Trigonometry & Calculus

Vector and Matrix Quantities N-VM

Represent and model with vector quantities.

1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\boldsymbol{v}, |\boldsymbol{v}|, ||\boldsymbol{v}||, v$).
2. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
3. (+) Solve problems involving velocity and other quantities that can be represented by vectors.

Perform operations on vectors.

4. (+) Add and subtract vectors.
    a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
    b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
    c. Understand vector subtraction $\boldsymbol{v} - \boldsymbol{w}$ as $\boldsymbol{v} + (-\boldsymbol{w})$, where $-\boldsymbol{w}$ is the additive inverse of $\boldsymbol{w}$, with the same magnitude as $\boldsymbol{w}$ and pointing in the opposite direction.

Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

5. (+) Multiply a vector by a scalar.
    a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.
    b. Compute the magnitude of a scalar multiple $c\boldsymbol{v}$ using $||c\boldsymbol{v}|| = |c|\boldsymbol{v}$. Compute the direction of $c\boldsymbol{v}$ knowing that when $|c|\boldsymbol{v} \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$).

Interpreting Functions F-IF

Understand the concept of a function and use function notation.

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Interpret functions that arise in applications in terms of the context.

3. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★
4. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.★
5. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

Building Functions F-BF

Build a function that models a relationship between two quantities.

1. Write a function that describes a relationship between two quantities.★
    a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
    b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model*.

c. (+) Compose functions. *For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.*

Build new functions from existing functions.

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k, k\,f(x), f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
4. Find inverse functions.
    a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example, $f(x) = 2\,x^3$ or $f(x) = (x + 1)/(x - 1)$ for $x \neq 1$.*
    b. (+) Verify by composition that one function is the inverse of another.
    c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
    d. (+) Produce an invertible function from a non-invertible function by restricting the domain.
5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Linear, Quadratic, and Exponential Models F-LE

Construct and compare linear, quadratic, and exponential models and solve problems.

4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Trigonometric Functions F-TF

Extend the domain of trigonometric functions using the unit circle.

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3, \pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x, \pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.
4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Model periodic phenomena with trigonometric functions.

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★
6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.★

Prove and apply trigonometric identities.

8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.
9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Similarity, Right Triangles, and Trigonometry G-SRT

Define trigonometric ratios and solve problems involving right triangles.

6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
7. Explain and use the relationship between the sine and cosine of complementary angles.
8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★

Apply trigonometry to general triangles

9. (+) Derive the formula $A = 1/2\ ab\ \sin(c)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
10. (+) Prove the Laws of Sines and Cosines and use them to solve problems.
11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Expressing Geometric Properties with Equations G-GPE

Translate between the geometric description and the equation for a conic section.

1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
2. Derive the equation of a parabola given a focus and directrix.
3. (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.