Trigonometry & Calculus

Grade Level:	10-12
Length:	1 Year
Period(s) Per Day:	1
Credit:	1
Credit Requirement Fulfilled:	Mathematics

Essential Understanding:

Students will investigate the underlying principles and applications of trigonometry. This course will review analytic geometry, study trigonometric functions and their use in modeling real-world phenomena, study trigonometric identities, study the principles and concepts of the derivative, and apply differential calculus to solve problems in a variety of areas including, but not limited to, physics and business. **This course can be dual credit with MATH 112 within the Montana University System.**

Theme Samples:

1. Algebraic Expressions, Functions, and Equations
2. Trigonometric Functions
3. Angles
4. Radian Measure
5. Circular Functions
6. Trigonometric Identities
7. Inverse Trigonometric Functions
8. Applications of Trigonometry and Vectors
9. Complex Numbers, Polar Equations, and Parametric Equations
10. Limits and Continuity
11. Derivatives

Course Objectives, Student Objectives, and Expectations:

Students will be able:

1. To review algebra, functions and graphs.
2. To identify angle relationships and solve both right triangles and oblique triangles.
3. To evaluate trigonometric functions and apply trigonometric functions.
4. To understand the relationship between degree measure and radian measure.
5. To manipulate and graph trigonometric functions.
6. To prove and apply trigonometric identities.
7. To solve trigonometric equations and use inverse trigonometric functions.
8. To use and apply the Law of Sines and Law of Cosines.
9. To manipulate and graph parametric equations and apply and use complex numbers.
10. To define and calculate limits of function values.
11. To understand derivatives, how they work, and discover differential calculus.

Pacing and Pertinent Montana Common Core Standards

Semester 1

Unit R—Algebra Review	F-IF, F-BF
Unit 1—Trigonometric Functions	F-IF, F-BF, F-TF
Unit 2—Acute Angles and Right Triangles	G-SRT
Unit 3—Radian Measure and the Unit Circle	F-TF, G-SRT
Unit 4—Graphs of the Circular Functions	F-TF
Unit 5—Trigonometric Identities	F-TF

Semester 2

Unit 6—Inverse Circular Functions and Trigonometric Equations	F-BF
Unit 7—Applications of Trigonometry and Vectors	N-VM
Unit 8—Complex Numbers and Polar and Parametric Equations	N-CN
Unit 9—Limits and Continuity	(Advanced, no CCSS)
Unit 10—Derivatives	(Advanced, no CCSS)

Course Outline and Assessments

1st Semester

Algebra Review
- A. Basic Concepts from Algebra
- B. Real Number Operations and Properties
- C. Exponents, Polynomials, and Factoring
- D. Rational Expressions
- E. Radical Expressions
- F. Equations and Inequalities
- G. Rectangular Coordinates and Graphs
- H. Functions
- I. Graphing Techniques

Trigonometric Functions
- A. Angles
- B. Angle Relationships and Similar Triangles
- C. Trigonometric Functions
- D. Using the Definitions of the Trigonometric Functions

Acute Angles and Right Triangles
- A. Trigonometric Functions of Acute Angles
- B. Trigonometric Functions of Non-Acute Angles
- C. Approximations of Trigonometric Function Values
- D. Solutions and Applications of Right Triangles
- E. Further Applications of Right Triangles

Radian Measure and the Unit Circle
- A. Radian Measure

B. Applications of Radian Measure
C. The Unit Circle and Circular Functions
D. Linear and Angular Speed

Graphs of the Circular Functions
A. Graphs of the Sine and Cosine Functions
B. Translations of the Graphs of the Sine and Cosine Functions
C. Graphs of the Tangent and Cotangent Functions
D. Graphs of the Secant and Cosecant Functions
E. Harmonic Motion

Trigonometric Identities
A. Fundamental Identities
B. Verifying Trigonometric Identities
C. Sum and Difference Identities for Cosine
D. Sum and Difference Identities for Sine and Tangent
E. Double-Angle Identities
F. Half-Angle Identities

2nd Semester

Inverse Circular Functions and Trigonometric Equations
A. Inverse Circular Functions
B. Trigonometric Equations I
C. Trigonometric Equations II
D. Equations Involving Inverse Trigonometric Functions

Applications of Trigonometry and Vectors
A. Oblique Triangles and the Law of Sines
B. The Ambiguous Case of the Law of Sines
C. The Law of Cosines
D. Geometrically Defined Vectors and Applications
E. Algebraically Defined Vectors and the Dot Product

Complex Numbers, Polar Equations, and Parametric Equations
A. Complex Numbers
B. Trigonometric (Polar) Form of Complex Numbers
C. The Product and Quotient Theorems
D. De Moivre's Theorem; Powers and Roots of Complex Numbers
E. Polar Equations and Graphs
F. Parametric Equations, Graphs, and Applications

Limits and Continuity
A. Rates of Change and Limits
B. Limits Involving Infinity
C. Continuity
D. Rates of Change and Tangent Lines

Derivatives
A. Derivative of a Function

B. Differentiability
C. Rules for Differentiation
D. Velocity and Other Rates of Change
E. Derivatives of Trigonometric Functions
F. Chain Rule

Timeline:

Unit R—Algebra Review	2 ½ weeks
Unit 1—Trigonometric Functions	1 ½ weeks
Unit 2—Acute Angles and Right Triangles	1 ½ weeks
Unit 3—Radian Measure and the Unit Circle	2 weeks
Unit 4—Graphs of the Circular Functions	3 weeks
Unit 5—Trigonometric Identities	3 weeks
Unit 6—Inverse Circular Functions and Trigonometric Equations	2 ½ weeks
Unit 7—Applications of Trigonometry and Vectors	2 ½ weeks
Unit 8—Complex Numbers and Polar and Parametric Equations	2 weeks
Unit 9—Limits and Continuity	3 ½ weeks
Unit 10—Derivatives	3 ½ weeks

Montana Standards for Trigonometry & Calculus

The Complex Number System N-CN

Represent complex numbers and their operations on the complex plane.

4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

Vector and Matrix Quantities N-VM

Represent and model with vector quantities.

1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\boldsymbol{v}, |\boldsymbol{v}|, ||\boldsymbol{v}||, v$).
2. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
3. (+) Solve problems involving velocity and other quantities that can be represented by vectors.

Perform operations on vectors.

4. (+) Add and subtract vectors.
    a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
    b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.

c. Understand vector subtraction $\boldsymbol{v} - \boldsymbol{w}$ as $\boldsymbol{v} + (-\boldsymbol{w})$, where $-\boldsymbol{w}$ is the additive inverse of $\boldsymbol{w}$, with the same magnitude as $\boldsymbol{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

5. (+) Multiply a vector by a scalar.
   a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.
   b. Compute the magnitude of a scalar multiple $c\boldsymbol{v}$ using $||c\boldsymbol{v}|| = |c|\boldsymbol{v}$. Compute the direction of $c\boldsymbol{v}$ knowing that when $|c|\boldsymbol{v} \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$).

Interpreting Functions F-IF

Understand the concept of a function and use function notation.

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Interpret functions that arise in applications in terms of the context.

3. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
4. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.
5. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Building Functions F-BF

Build a function that models a relationship between two quantities.

1. Write a function that describes a relationship between two quantities.
   a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
   b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model*.
   c. (+) Compose functions. *For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as*

a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.

Build new functions from existing functions.

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k, k\,f(x), f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
4. Find inverse functions.
    a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2x^3$ *or* $f(x) = (x + 1)/(x - 1)$ *for* $x \neq 1$.
    b. (+) Verify by composition that one function is the inverse of another.
    c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
    d. (+) Produce an invertible function from a non-invertible function by restricting the domain.
5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Linear, Quadratic, and Exponential Models F-LE

Construct and compare linear, quadratic, and exponential models and solve problems.

4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Trigonometric Functions F-TF

Extend the domain of trigonometric functions using the unit circle.

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3, \pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x, \pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.
4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Model periodic phenomena with trigonometric functions.

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

Prove and apply trigonometric identities.

8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.
9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Similarity, Right Triangles, and Trigonometry G-SRT

Define trigonometric ratios and solve problems involving right triangles.

6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
7. Explain and use the relationship between the sine and cosine of complementary angles.
8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Apply trigonometry to general triangles

9. (+) Derive the formula $A = 1/2\ ab\ \sin(c)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
10. (+) Prove the Laws of Sines and Cosines and use them to solve problems.
11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Resources:

Textbook: Trigonometry, 12th Edition
Textbook Resources: Trigonometry, 12th Edition, Teacher Resources
Textbook: Calculus—Graphical, Numerical, Algebraic
HPS Technology Standards
Montana Common Core State Standards
Desmos.com
DeltaMath.com