

Geometry

Grade Level:	9 (with Recommendation), 10, 11, 12
Length:	1 Year
Period(s) Per Day:	1
Credit:	1
Credit Requirement Fulfilled:	Mathematics

Course Description

This course covers the main theories and postulates of Euclidean Geometry, which pertain to parallel lines, triangles, polygons, and circles. This will include studying angles and other geometric concepts and how to prove these concepts.

Theme Samples

1. Visualization
2. Transformations
3. Measurement
4. Reasoning and Proof
5. Similarity
6. Coordinate Geometry

Course Objectives and Expectations

1. To see relationships between two figures and help you connect properties of real objects with two-dimensional drawings of these objects.
2. To transform mathematical functions that model relationships with figures.
3. To apply some attributes of geometric figures, such as length, area, volume, and angle measure.
4. To verify complex truths by reasoning from simpler truths using deductive reasoning.
5. To understand some complex truths using a coordinate plane along with distance, midpoint, and slope formulas.

Pacing

Montana Common Core Standard

Semester 1

Unit 1- Tools of Geometry
Unit 2- Reasoning and Proof

G-MG
G-CO

Unit 3- Parallel and Perpendicular Lines	G-GPE
Unit 4- Congruent Triangles	G-CO
Unit 5- Relationships with in Triangles	G-CO
Unit 6- Polygons and Quadrilaterals	G-CO
Unit 7- Similarity	G-SRT

1st Semester

Tools of Geometry

- A. Nets and Drawings for Visualizing Geometry
- B. Points, Lines, and Planes
- C. Measuring Segments
- D. Measuring Angles
- E. Exploring Angle Pairs
- F. Basic Constructions
- G. Midpoint and Distance in a Coordinate Plane
- H. Perimeter, Circumference, and Area

Reasoning and Proof

- A. Patterns and Inductive Reasoning
- B. Conditional Statements
- C. Bi-conditionals and Definitions
- D. Deductive Reasoning
- E. Reasoning in Algebra and Geometry
- F. Proving Angles Congruent

Parallel and Perpendicular Lines

- A. Lines and Angles
- B. Properties of Parallel lines
- C. Proving Lines Parallel
- D. Parallel and Perpendicular Lines
- E. Parallel Lines and Triangles
- F. Constructing Parallel and Perpendicular Lines
- G. Equations of Lines
- H. Slopes of Parallel and Perpendicular Lines

Congruent Triangles

- A. Congruent Figures
- B. SSS and SAS Congruence
- C. ASA and AAS Congruence
- D. Using Corresponding Parts of Congruent Triangles
- E. Isosceles and Equilateral Triangles
- F. Congruence in Right Triangles
- G. Congruence in Overlapping Triangles

Relationships within Triangles

- A. Midsegments of Triangles
- B. Perpendicular and Angle Bisectors
- C. Bisectors of Triangles
- D. Medians and Altitudes
- E. Indirect Proof
- F. Inequalities in one Triangle
- G. Inequalities in two Triangles

Polygons and Quadrilaterals

- A. Polygon Angle Sum Theorem
- B. Properties of Parallelograms
- C. Proving Parallelograms
- D. Properties of Rhombuses, Rectangles, and Squares
- E. Conditions of Rhombuses, Rectangles, and Squares
- F. Trapezoids and Kites
- G. Polygons in the Coordinate Plane
- H. Applying Coordinate Geometry
- I. Proofs using Coordinate Geometry

Similarity

- A. Ratios and Proportions
- B. Similar Polygons
- C. Proving Triangles Similar
- D. Similarity in Right Triangles
- E. Proportions in Triangles

Semester 2

Unit 8- Right Triangles and Similarity

G-SRT

Unit 9- Transformations

G-CO

Unit 10- Area

G-GMD

Unit 11- Surface Area and Volume

G-GMD

Unit 12- Circles

G-C

2nd Semester

Right Triangles and Trigonometry

- A. Pythagorean Theorem and its Converse
- B. Special Right Triangles
- C. Trigonometry
- D. Angles of Elevation and Depression
- E. Vectors

Transformations

- A. Translations

- B. Reflections
- C. Rotations
- D. Symmetry
- E. Dilations
- F. Compositions of Reflections
- G. Tessellations

Area

- A. Parallelograms and Triangles
- B. Trapezoids, Kites, and Rhombuses
- C. Regular Polygons
- D. Perimeters and Areas of Similar Figures
- E. Trigonometry and Area
- F. Circles and Arcs
- G. Areas of Circles and Sectors
- H. Geometric Probability

Surface Area and Volume

- A. Space Figures and Cross Sections
- B. Prisms and Cylinders
- C. Pyramids and Cones
- D. Spheres
- E. Area and Volume of Similar Solids

Circles

- A. Tangent Lines
- B. Chords and Arcs
- C. Inscribed Angles
- D. Angle measure and Segment Length
- E. Circles in the Coordinate Plane
- F. Locus: Set of Points

Timeline

Unit 1	(2 weeks to cover)
Unit 2	(2 week to cover)
Unit 3	(2 ½ weeks to cover)
Unit 4	(2 ½ weeks to cover)
Unit 5	(2 ½ weeks to cover)
Unit 6	(2 weeks to cover)
Unit 7	(2 weeks to cover)
Unit 8	(2 ½ weeks to cover)
Unit 9	(2 weeks to cover)
Unit 10	(3 weeks to cover)
Unit 11	(3 weeks to cover)

Montana Standards for Geometry**Congruence G-CO**

Experiment with transformations in the plane.

1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. Understand congruence in terms of rigid motions.

6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. Prove geometric theorems.

9. Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

10. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180° ; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

11. Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and

conversely, rectangles are parallelograms with congruent diagonals. Make geometric constructions.

12. Make formal geometric constructions, including those representing Montana American Indians, with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Similarity, Right Triangles, and Trigonometry G-SRT

Understand similarity in terms of similarity transformations.

1. Verify experimentally the properties of dilations given by a center and a scale factor: a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. Prove theorems involving similarity.

4. Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.

5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. Define trigonometric ratios and solve problems involving right triangles.

6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

7. Explain and use the relationship between the sine and cosine of complementary angles.

8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. Apply trigonometry to general triangles.

9. (+) Derive the formula $A = \frac{1}{2} ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

10. (+) Prove the Laws of Sines and Cosines and use them to solve problems.

11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Circles G-C

Understand and apply theorems about circles.

1. Prove that all circles are similar.
2. Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
4. (+) Construct a tangent line from a point outside a given circle to the circle. Find arc lengths and areas of sectors of circles.
5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Expressing Geometric Properties with Equations G-GPE

Translate between the geometric description and the equation for a conic section.

1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
2. Derive the equation of a parabola given a focus and directrix.
3. (+) Derive the equations of ellipses and hyperbolas given the foci and directrices. Use coordinates to prove simple geometric theorems algebraically.
4. Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point $(0, 2)$.
5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. ★

Geometric Measurement and Dimension G-GMD

Explain volume formulas and use them to solve problems.

1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.
 2. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.
 3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. ★
- Visualize relationships between two-dimensional and three-dimensional objects. 4. Identify the

shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Modeling with Geometry G-MG

Apply geometric concepts in modeling situations.

1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder; modeling a Montana American Indian tipi as a cone).★

2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★

3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★