

## **Geometry**

Grade Level:	9 (with Recommendation), 10, 11, 12
Length:	1 Year
Period(s) Per Day:	1
Credit:	1
Credit Requirement Fulfilled:	Mathematics, Required

### **Course Description:**

This course covers the main theories and postulates of Euclidean Geometry, which pertain to parallel lines, triangles, polygons, and circles. This will include studying angles and other geometric concepts and how to prove these concepts.

### **Theme Samples:**

1. Visualization
2. Transformations
3. Measurement
4. Reasoning and Proof
5. Similarity
6. Coordinate Geometry

### **Course Objectives and Expectations:**

Students will be able:

1. To see relationships between two figures and help you connect properties of real objects with two-dimensional drawings of these objects.
2. To transform mathematical functions that model relationships with figures.
3. To apply some attributes of geometric figures, such as length, area, volume, and angle measure.
4. To verify complex truths by reasoning from simpler truths using deductive reasoning.
5. To understand some complex truths using a coordinate plane along with distance, midpoint, and slope formulas.

### **Pacing and Montana Common Core Standard:**

Semester 1

Unit 1- Basics of Geometry

G-MG

Unit 2- Reasoning and Proof

G-CO

Unit 3- Parallel and Perpendicular Lines

G-GPE

Unit 4- Transformations	G-CO
Unit 5- Congruent Triangles	G-CO
Unit 6- Relationships with in Triangles	G-CO
Semester 2	
Unit 7- Quadrilaterals and Other Polygons	G-SRT
Unit 8-Similarity	G-SRT
Unit 9- Right Triangles and Trigonometry	G-SRT
Unit 10- Circles	G-C
Unit 11- Circumference, Area, and Volume	G-GMD
Unit 12- Probability	

## Course Outline and Assessments

### 1<sup>st</sup> Semester

#### Basics of Geometry

- A. Points, Lines, and Planes
- B. Measuring and Constructions
- C. Using Midpoint and Distance Formula
- D. Perimeter and Area in the Coordinate Plane
- E. Measuring and Constructing Angles
- F. Describing Pairs of Angles

#### Reasoning and Proof

- A. Conditional Statements
- B. Inductive and Deductive Reasoning
- C. Postulates and Diagrams
- D. Algebraic Reasoning
- E. Proving Statements about Segments and Angles
- F. Proving Geometric Relationships

#### Parallel and Perpendicular Lines

- A. Pairs of Lines and Angles
- B. Parallel lines and Transversals
- C. Proofs with Parallel Lines
- D. Proofs with Perpendicular Lines
- E. Equations of Parallel and Perpendicular Lines

#### Transformations

- A. Translations
- B. Reflections
- C. Rotations
- D. Congruence and Transformations
- E. Dilations
- F. Similarity and Transformations

#### Congruent Triangles

- A. Angles of Triangles

- B. Congruent Polygons
- C. Proving Triangles Congruent by SAS
- D. Equilateral and Isosceles Triangles
- E. Proving Triangles Congruent by SSS
- F. Proving Triangles Congruent by ASA and AAS
- G. Using Congruent Triangles
- H. Coordinate Proofs

#### Relationships with Triangles

- A. Perpendicular and Angle Bisectors
- B. Bisectors of Triangles
- C. Medians and Altitudes of Triangles
- D. The Triangle Midsegment Theorem
- E. Indirect Proof and Inequalities in One Triangle
- F. Inequalities in Two Triangles

#### 2<sup>nd</sup> Semester

##### Quadrilaterals and Other Polygons

- A. Angles of Polygons
- B. Properties of Parallelograms
- C. Proving That a Quadrilateral is a Parallelogram
- D. Properties of Special Parallelograms
- E. Properties of Trapezoids and Kites

##### Similarity

- A. Similar Polygons
- B. Proving Triangles Similarity by AA
- C. Proving Triangles Similarity by SSS and SAS
- D. Proportionality Theorems

##### Right Triangles and Trigonometry

- A. The Pythagorean Theorem
- B. Special Right Triangles
- C. Similar Right Triangles
- D. The Tangent Ratio
- E. The Sine and Cosine Ratios
- F. Solving Right Triangles
- G. Law of Sines and Law of Cosines

##### Circles

- A. Lines and Segments That Intersect Circles
- B. Finding Arc Measures
- C. Using Chords
- D. Inscribed Angles and Polygons
- E. Angles Relationships in Circles
- F. Segment Relationships in Circles
- G. Circles in the Coordinate Plane

## Circumference, Area, and Volume

- A. Circumference and Arc Length
- B. Areas of Circles and Sectors
- C. Areas of Polygons
- D. Three-Dimensional Figures
- E. Volumes of Prisms and Cylinders
- F. Volumes of Pyramids
- G. Surface Areas and Volumes of Cones
- H. Surface Areas and Volumes of Spheres

## Probability (If time allows, but not on semester exam)

- A. Sample Spaces and Probability
- B. Independent and Dependent Events
- C. Two-Way Tables and Probability
- D. Probability of Disjoint and Overlapping Events
- E. Permutations and Combinations
- F. Binomial Distribution

## **Timeline:**

Unit 1	(2 weeks to cover)
Unit 2	(2 week to cover)
Unit 3	(2 ½ weeks to cover)
Unit 4	(2 ½ weeks to cover)
Unit 5	(2 ½ weeks to cover)
Unit 6	(2 weeks to cover)
Unit 7	(2 weeks to cover)
Unit 8	(2 ½ weeks to cover)
Unit 9	(2 weeks to cover)
Unit 10	(3 weeks to cover)
Unit 11	(3 weeks to cover)
Unit 12	(2 weeks to cover)

## **Montana Standards for Geometry**

### **Congruence G-CO**

Experiment with transformations in the plane.

1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give

other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. Understand congruence in terms of rigid motions.

6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. Prove geometric theorems.

9. Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

10. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to  $180^\circ$ ; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

11. Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. Make geometric constructions.

12. Make formal geometric constructions, including those representing Montana American Indians, with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

### **Similarity, Right Triangles, and Trigonometry G-SRT**

Understand similarity in terms of similarity transformations.

1. Verify experimentally the properties of dilations given by a center and a scale factor: a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves

a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. Prove theorems involving similarity.

4. Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.

5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. Define trigonometric ratios and solve problems involving right triangles.

6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

7. Explain and use the relationship between the sine and cosine of complementary angles.

8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. Apply trigonometry to general triangles.

9. (+) Derive the formula  $A = \frac{1}{2} ab \sin(C)$  for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

10. (+) Prove the Laws of Sines and Cosines and use them to solve problems.

11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

## **Circles G-C**

Understand and apply theorems about circles.

1. Prove that all circles are similar.

2. Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

4. (+) Construct a tangent line from a point outside a given circle to the circle. Find arc lengths and areas of sectors of circles.

5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

## **Expressing Geometric Properties with Equations G-GPE**

Translate between the geometric description and the equation for a conic section.

1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

2. Derive the equation of a parabola given a focus and directrix.
3. (+) Derive the equations of ellipses and hyperbolas given the foci and directrices. Use coordinates to prove simple geometric theorems algebraically.
4. Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point  $(1, \sqrt{3})$  lies on the circle centered at the origin and containing the point  $(0, 2)$ .
5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

### **Geometric Measurement and Dimension G-GMD**

Explain volume formulas and use them to solve problems.

1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.
2. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.
3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. Visualize relationships between two-dimensional and three-dimensional objects.
4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

### **Modeling with Geometry G-MG**

Apply geometric concepts in modeling situations.

1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder; modeling a Montana American Indian tipi as a cone).
2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

### **Resources:**

Textbook: Glencoe Geometry

Textbook Resources: Glencoe Geometry, Teacher Resources

aleks.com