Contemporary Mathematics

Grade Level:	11, 12
Length:	1 Year
Period(s) Per Day:	1
Credit:	1
Credit Requirement Fulfilled:	Mathematics

Course Description:

This course surveys some of the important ideas and practical applications in mathematics and uses a variety of mathematical skills and technology to solve real problems. Topics include problem solving, financial math, mathematical modeling, and elementary statistics. **This course can be dual credit with MATH 105 within the Montana University System.**

Theme Samples:

1. Logic and Problem Solving
2. Numbers in the Real World
3. Managing Money
4. Statistical Reasoning
5. Putting Statistics to Work
6. Probability
7. Exponential and Linear Functions
8. Modeling
9. Mathematics and the Arts
10. Mathematics and Politics

Course Objectives, Student Objectives, and Expectations:

Students will be able:

1. To apply the basic principles of unit analysis.
2. To become familiar with subtle uses and abuses of percentages.
3. To apply the basic principles of financial budgeting.
4. To understand how statistical studies are conducted and how to evaluate statistical claims.
5. To characterize data and make inferences from data.
6. To explore the basic concepts of probability and use probability to describe risk.
7. To distinguish between linear growth/decay and exponential growth/decay.
8. To apply mathematical functions to model real-world phenomena.
9. To investigate examples that use geometry to solve problems that arise in everyday life.
10. To identify how mathematics and politics have been part of human culture for millennia.

Pacing and Pertinent Montana Common Core Standard

Semester 1

Unit 1—Approaches to Problem Solving	N-Q
Unit 2—Numbers in the Real World	F-IF
Unit 3—Managing Money	(Advanced, no CCSS)
Unit 4—Statistical Reasoning	S-ID
Unit 5—Putting Statistics to Work	S-ID, S-IC
Unit 6—Probability: Living with the Odds	S-CP

Semester 2

Unit 7—Exponential Astonishment	A-CED, F-LE
Unit 8—Modeling Our World	F-LE, F-IF, F-BF
Unit 9—Modeling with Geometry	G-MG
Unit 10—Mathematics and the Arts	(Advanced, no CCSS)
Unit 11—Mathematics and Politics	(Advanced, no CCSS)

Course Outline and Assessments

1st Semester

- Approaches to Problem Solving
  - A. Working with Units
  - B. Problem Solving with Units
  - C. Problem Solving Guidelines and Hints
- Numbers in the Real World
  - A. Uses and Abuses of Percentages
  - B. Putting Numbers in Perspective
  - C. Dealing with Uncertainty
  - D. Index Numbers: The CPI and Beyond
  - E. How Numbers Can Deceive: Polygraphs, Mammograms, and More
- Managing Money
  - A. Taking Control of Your Finances
  - B. The Power of Compounding
  - C. Savings Plans and Investments
  - D. Loan Payments, Credit Cards, and Mortgages
  - E. Income Taxes
  - F. Understanding the Federal Budget
- Statistical Reasoning
  - A. Fundamentals of Statistics
  - B. Should You Believe a Statistical Study?
  - C. Statistical Tables and Graphs
  - D. Graphics in the Media
  - E. Correlation and Causality
- Putting Statistics to Work
  - A. Characterizing Data
  - B. Measures of Variation
  - C. The Normal Distribution

D. Statistical Inference

Probability: Living with the Odds

A. Fundamentals of Probability
B. Combining Probabilities
C. The Law of Large Numbers
D. Assessing Risk
E. Counting and Probability

2nd Semester

Exponential Astonishment

A. Growth: Linear versus Exponential
B. Doubling Time and Half-Life
C. Real Population Growth
D. Logarithmic Scales: Earthquakes, Sounds, and Acids

Modeling Our World

A. Functions: The Building Blocks of Mathematical Models
B. Linear Modeling
C. Exponential Modeling

Modeling with Geometry

A. Fundamentals of Geometry
B. Problem Solving with Geometry
C. Fractal Geometry

Mathematics and the Arts

A. Mathematics and Music
B. Perspective and Symmetry
C. Proportion and the Golden Ratio

Mathematics and Politics

A. Voting: Does the Majority Always Rule?
B. Theory of Voting
C. Apportionment: The House of Representatives and Beyond
D. Dividing the Political Pie

Timeline:

Unit 1—Approaches to Problem Solving	1 ½ weeks
Unit 2—Numbers in the Real World	3 weeks
Unit 3—Managing Money	2 ½ weeks
Unit 4—Statistical Reasoning	3 weeks
Unit 5—Putting Statistics to Work	2 ½ weeks
Unit 6—Probability: Living with the Odds	2 ½ weeks
Unit 7—Exponential Astonishment	2 ½ weeks
Unit 8—Modeling Our World	3 weeks
Unit 9—Modeling with Geometry	3 ½ weeks
Unit 10—Mathematics and the Arts	3 ½ weeks
Unit 11—Mathematics and Politics	3 weeks

Montana Standards for Contemporary Mathematics

Quantities N-Q

Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
2. Define appropriate quantities for the purpose of descriptive modeling.
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Creating Equations A-CED

Create equations that describe numbers or relationships.

1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

Interpreting Functions F-IF

Analyze functions using different representations.

1. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
    a. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t, y = (0.97)^t, y = (1.01)^t, y = (1.2)^{\frac{t}{10}}$, *and classify them as representing exponential growth or decay.*
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Interpret functions that arise in applications in terms of the context.

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function* $h(n)$ *gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations.

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

Building Functions F-BF

Build a function that models a relationship between two quantities.

1. Write a function that describes a relationship between two quantities.

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model*.
c. (+) Compose functions. *For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.*

Build new functions from existing functions.

5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Linear, Quadratic, and Exponential Models F-LE

Construct and compare linear, quadratic, and exponential models and solve problems.

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
   a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
   b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
   c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Interpret expressions for functions in terms of the situation they model.

5. Interpret the parameters in a linear or exponential function in terms of a context.

Interpreting Categorical and Quantitative Data S-ID

Summarize, represent, and interpret data on a single count or measurement variable.

1. Represent data with plots on the real number line (dot plots, histograms, and box plots).
2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

Summarize, represent, and interpret data on two categorical and quantitative variables.

5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested*

by the context. Emphasize linear, quadratic, and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for a scatter plot that suggests a linear association.

Interpret linear models.

7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
8. Compute (using technology) and interpret the correlation coefficient of a linear fit.
9. Distinguish between correlation and causation.

Making Inferences and Justifying Conclusions S-IC

Understand and evaluate random processes underlying statistical experiments.

1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
6. Evaluate reports based on data.

Conditional Probability and the Rules of Probability S-CP

Understand independence and conditional probability and use them to interpret data.

1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).
2. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

7. Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.

Modeling with Geometry G-MG

Apply geometric concepts in modeling situations.

1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

Resources:
Textbook: Using & Understanding Mathematics—A Quantitative Reasoning Approach
Textbook Resources: Using & Understanding Mathematics—A Quantitative Reasoning Approach, Teacher Resources
HPS Technology Standards
Montana Common Core State Standards